

7.1: Introduction

Engineering is problem solving. Specifically, engineering is the application of practical and scientific knowledge to a [methodical](#) problem solving process. Unit 1 described a commonly used design process, but only briefly touched on the types of knowledge engineers need to utilize it effectively. This unit will provide budding robot designers with a beginners look at two topics helpful for solving problems in this field. To get more in-depth information on these topics, students should consider pursuing higher education in an engineering or science field.

7.2: Classical Mechanics

The field of Classical [Mechanics](#) deals with the study of bodies in motion, specifically the physical laws that govern bodies under the influence of forces. Much of the mechanical aspects of robotics design are heavily tied to the principles of this field. This unit will describe a few key concepts of Classical Mechanics that are particularly applicable.

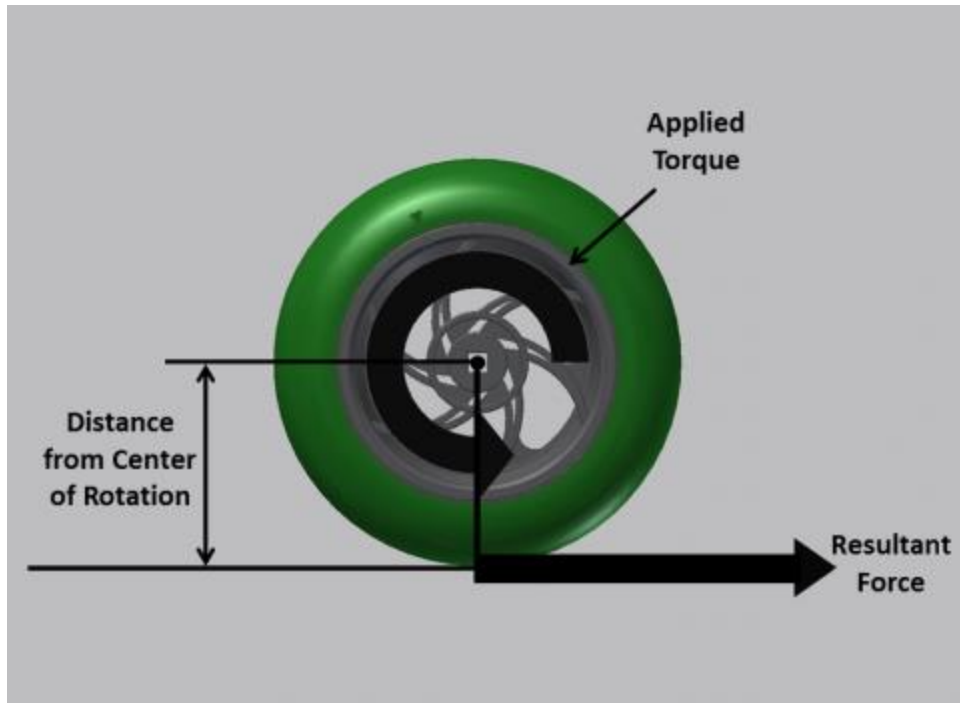
SPEED – A measure of how fast an object is moving. Describes a change in position with time (or more simply put, how far an object will travel over a given period of time.) This measure is given in units of distance per time (i.e. miles per hour, or feet per second.)

ROTATIONAL SPEED – Speed can also be expressed rotationally. This refers to how fast something is moving in a circle. It is measured in units of angular-distance per time (i.e. degrees per second) or rotational cycles per time (i.e. revolutions per minute.) When someone talks about “RPM” they are referencing rotational speed. When talking about the RPM of a car engine, one is describing how fast the engine is spinning.

ACCELERATION – A change in speed over a period of time is described as acceleration; the higher the acceleration the faster the change in speed. If a car goes from 0 miles per hour to 60 miles per hour in 2 seconds, it is a higher acceleration than if the car goes from 0 MPH to 40 MPH in 2 seconds. Acceleration is a rate of change of speed. No change means no acceleration – if something is moving at constant speed it is not accelerating.

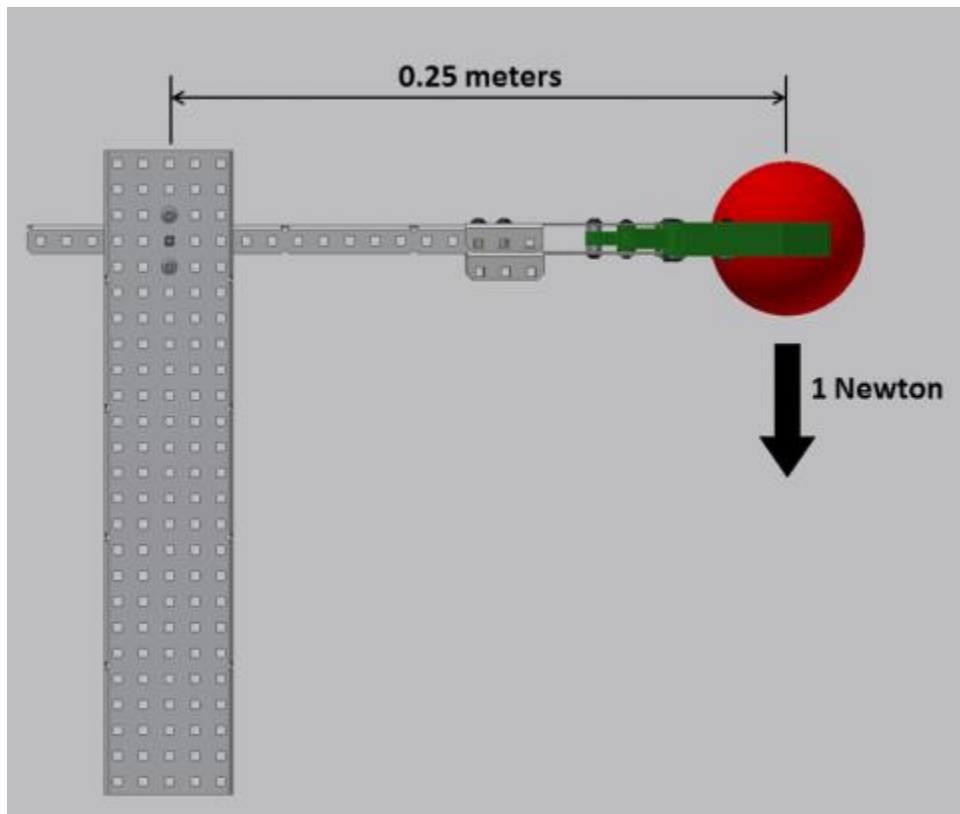
FORCE - Accelerations are caused by forces; they are influences that cause a change of movement, direction or shape. When one presses on an object, they are exerting a force on it. When a robot is accelerating, it does so because of the force its wheels exert on the floor. Force is measured in units such as Pounds or Newtons. For instance, the weight of an object is the force on the object due to gravity (accelerating the object towards the center of the earth.)

TORQUE – Force directed in a circle (rotating an object) is known as torque. Torque is a spinning force. If torque is spinning an object, the object will create a linear force at its edge. In the instance of a wheel spinning on the ground, the torque applied to the wheel axle creates a linear force at the edge of the tire where it contacts the ground. This is how one defines torque, a linear force at the edge of a circle. Torque is described by the [magnitude](#) of the force multiplied by the distance it is from the center of rotation (Force x Distance = Torque). Torque is measured in units of force*distance, such as Inch-Pounds or Newton-Meters.



In the above example of a wheel on a surface, if we know how much torque is applied to an axle with a wheel on it, we can find out how much force the wheel is applying on the floor. In this case, the wheel radius would be the distance the force is from the center of rotation.

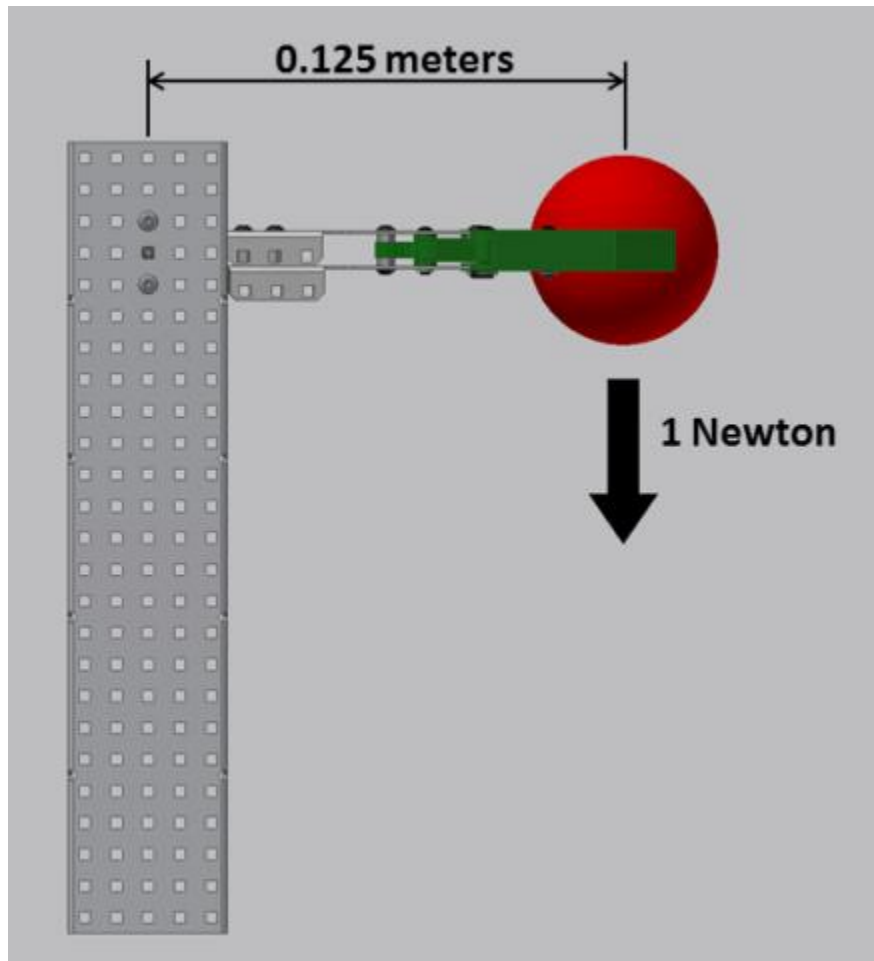
$$\text{Force} = \text{Torque} / \text{Wheel Radius}$$



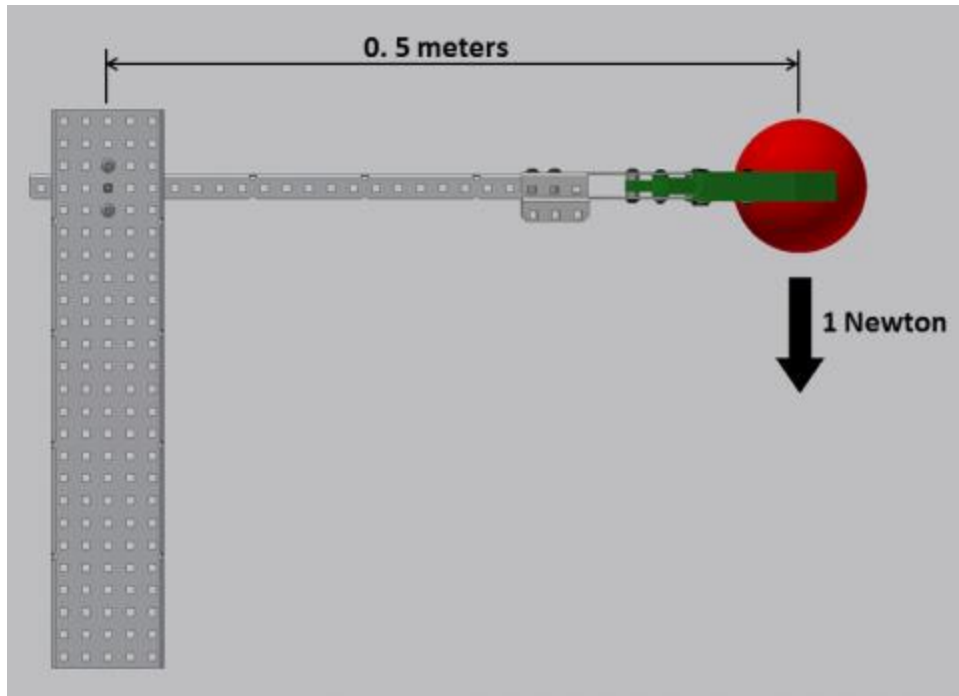
In the above example of a robot arm holding an object, we can calculate the torque required to lift the object. Since the object has a weight of 1 Newton, and the arm is 0.25 meter long (the object is 0.25 meters from the center of rotation), then

Torque = Force x Distance = 1 Newton x 0.25 Meter = 0.25 Newton-Meters.

This means the torque required to hold the object stationary is 0.25 Newton-Meters. In order to move it upwards the robot needs to apply MORE torque than 0.25 Newton-Meters to overcome gravity. The more torque the robot has, the more force it exerts on the object, the greater the acceleration on the object, and the faster the arm will lift it up.



Example 7.2



Example 7.3

For these examples, we can calculate the torque required to lift these objects as well.

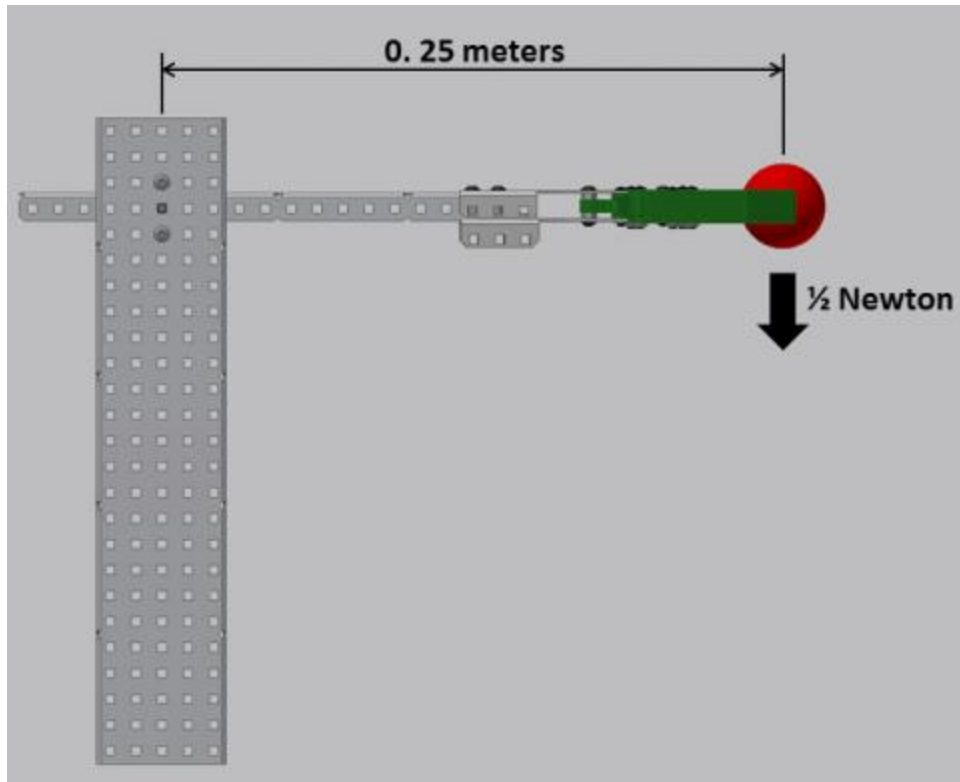
Example 7.2 – Torque = Force x Distance = 1 Newton x 0.125 meters = 0.125 Newton-Meters.

In this example, the arm is half the length of Example 1, and the torque required is also half. The arm length is proportional to the torque required; shorter arms require less torque to lift the same object as longer arms.

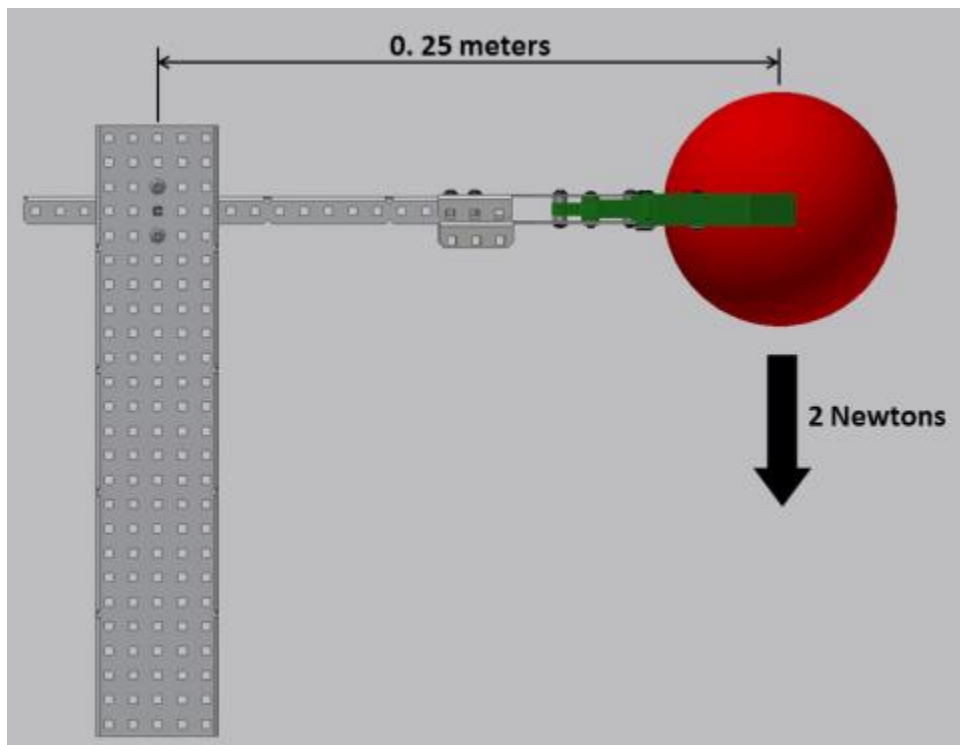
Example 7.3 – Torque = Force * Distance = 1 Newton x 0.5 meters = 0.5 Newton-Meters.

In this example, the arm length is double the length of Example 1, and the torque required is also double.

Another way to think of this is if a robot has limited torque at its arm joint, a shorter arm will be able to lift more weight than a long arm (but it can't lift it as high.)



Example 7.4



Example 7.5

These examples have the same robot arm lifting objects of different weights. How does this affect the required arm torque?

Example 4 – Torque = Force x Distance = ½ Newton x 0.25 meters = 0.125 Newton-Meters

Example 5 – Torque = Force x Distance = 2 Newton x 0.25 meters = 0.5 Newton-Meters

In these examples, as the weight of the object goes down, the torque required goes down as well. The applied weight is proportional to the torque required to lift it; heavier objects require more torque to lift.

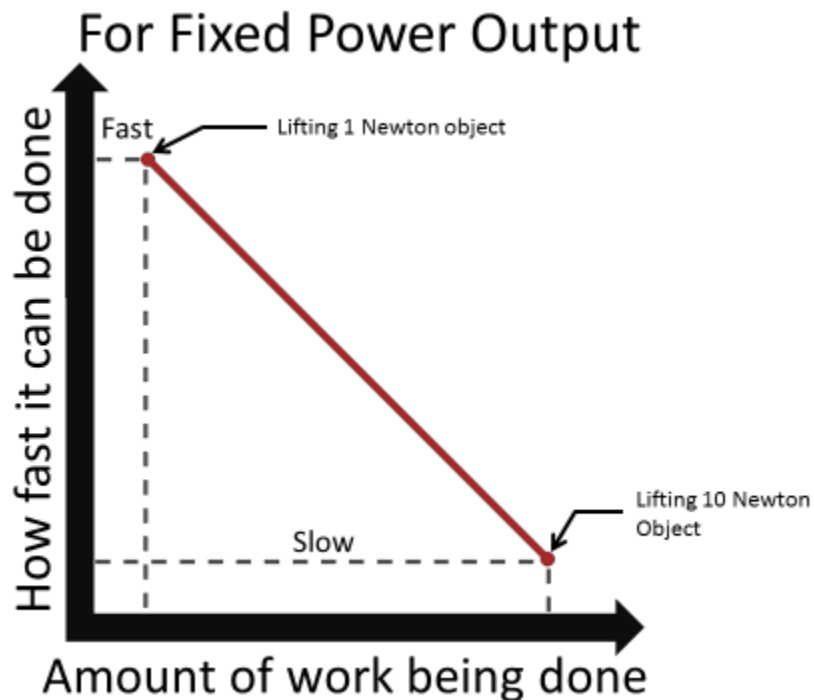
Budding robotics designers should note these key relationships between torque, arm length, and object weight.

WORK – The measure of force exerted over a distance is referred to as work. For instance, say it takes 10 pounds of force to hold an object. It would then take a certain amount of work to lift this object 10 inches, and it would take double that work to lift it 20 inches. Work can also be thought of as a change in energy.

POWER - Most people are more familiar with Power as an electrical term, but it is part of mechanical physics as well.

Power is defined as the **RATE** that work is performed. How fast can someone do their work?

In robotics design it is handy to think of power as a limit, since competition robotic systems are limited in the amount of power they can output. If a robot needs to lift a 2 Newton weight (exerting a 2 Newton force) the amount of power the robot can output limits how **FAST** (the rate) at which the robot can lift it. If the robot is capable of outputting lots of power, it will be able to lift it quickly. If it can only output a small amount of power, it will lift it slowly (or not at all!)



Power is defined as Force multiplied by Velocity (how fast one can push with a constant force), and is frequently expressed in units of Watts.

Power [Watts] = Force [Newtons] x Velocity [Meters / Second]

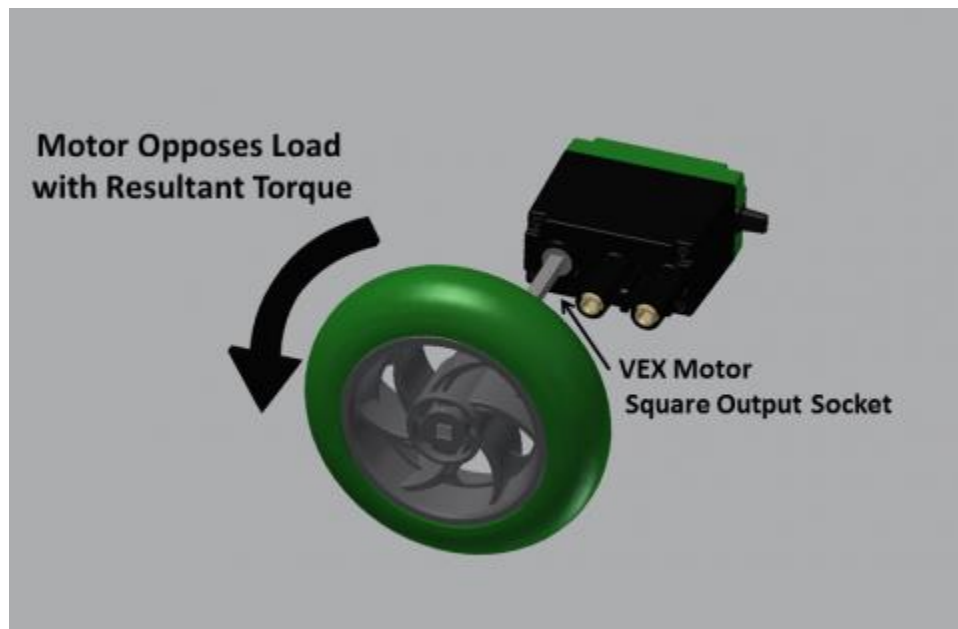
1 Watt = 1 (Newton x Meter) / Second

So how does this apply to competition robotics? Robot designers have certain power limitations they must remain below. Competition robotics designers using the VEX Robotics Design also need to consider the physical limitations presented by the motors. These motors have limited power, and they can only do so much work, so fast.

Note: these are basic descriptions of very advanced concepts. For more in-depth discussions of these physical properties, students should pursue higher education in the STEM fields.

7.3: DC Motors

Actuators are mechanisms used to act upon an environment, usually for moving or controlling a mechanism or system. Actuators drive everything that moves on a competition robot. The most common type of actuator in this application is a motor; in particular, VEX Robots utilize Direct Current (DC) Motors.



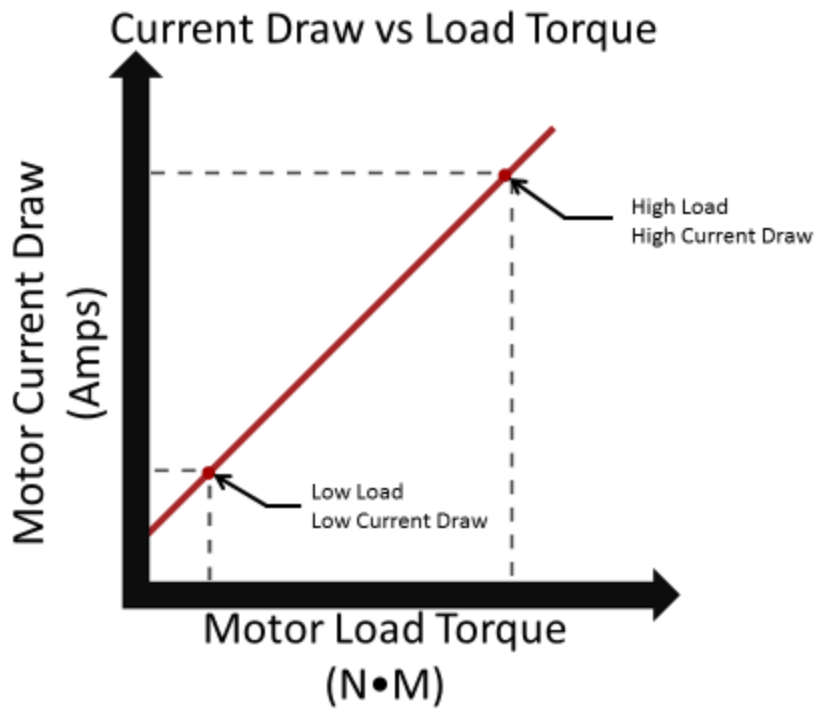
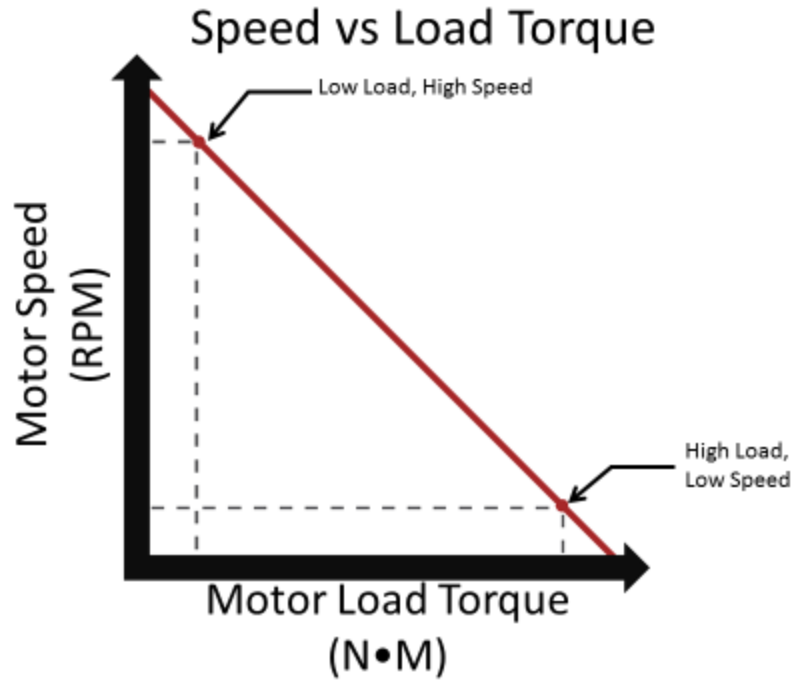
Motors convert electrical energy into mechanical energy through the use of electro-magnetic fields, and rotating wire coils. When a [voltage](#) is applied to a motor it outputs a fixed amount of mechanical power. The mechanical power is seen as the motor's output (usually some shaft, socket, or gear), spinning at some [speed](#) with some amount of [torque](#).

Motor Loading

Motors only apply torque in response to loading. Ideally, with no loading on the output the motor will spin very, very fast with no torque. This never happens in real life, since there is always friction in the motor system acting as a [load](#) and requiring the motor to output torque to overcome it. The higher the load placed on the motor output, the more the motor will "fight back" with an opposing torque. However, since the motor outputs a fixed amount of power, the more torque the motor outputs, the slower its [rotational speed](#). The more work one makes the motor do, the slower it spins. If one keeps increasing the load on the motor, eventually the load overcomes the motor and it stops spinning. This is called a STALL.

Current Draw

The motor draws a certain amount of electrical [current](#) (expressed in units of amps) depending on how much load is placed on it. As the load increases on the motor, the more torque the motor outputs to overcome it and the more current the motor draws.



As seen in the graphs above, current and torque load are proportional. More torque load means more current draw, but current and rotational speed are inverse. The faster the motor spins, the less current it draws.

The “key” Motor Characteristics

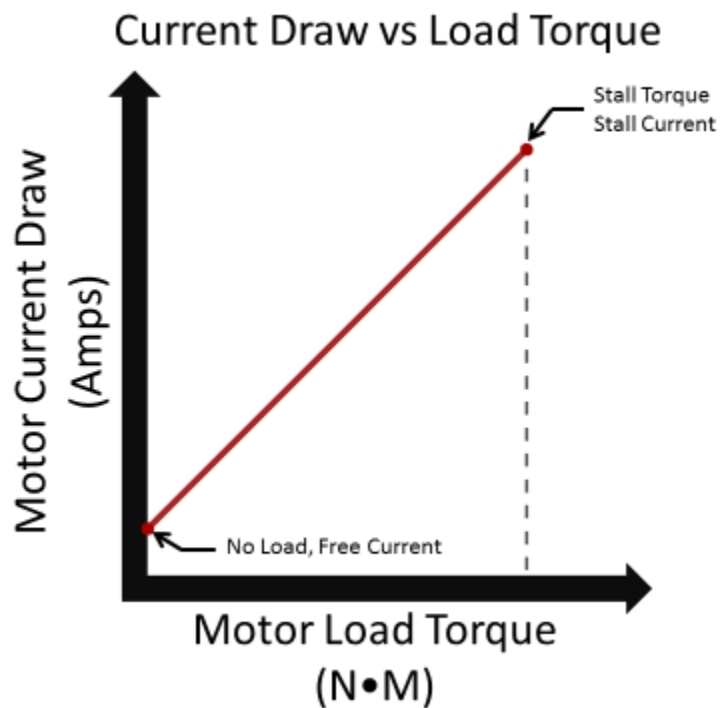
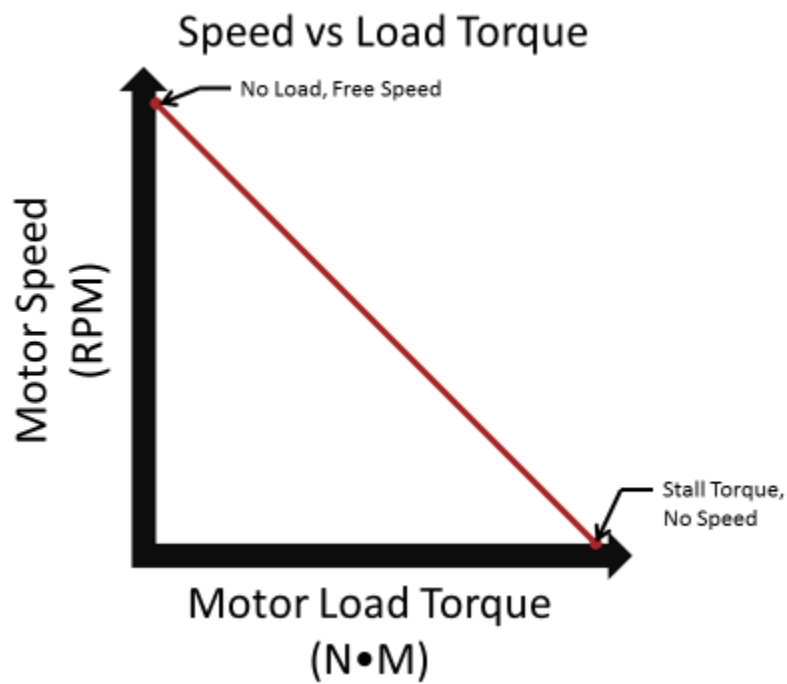
All motors are different, and their properties vary depending on their type, configuration, and manufacture. There are four main characteristics that govern the types of DC motors used commonly in competition robotics.

Stall Torque (N-m) – The amount of load placed on a motor that will cause it to stop moving.

Free Speed (RPM) – The maximum rotational speed a motor will run at when it is under no load.

Stall Current (Amp) – the amount of current a motor will draw when it is stalled.

Free Current (Amp) – The amount of current a motor will draw when it is under no load.



Based on the above relationships, one can see how the concept of power comes into play. With a given loading, the motor can only spin at a certain speed.

Since the relationships shown above are linear and proportional, it is a simple matter of plotting the torque-speed and torque-current graphs for any motor by experimentally determining two points on each graph.

Varying Power with Voltage

The Power output of a DC Motor varies with the voltage applied. This means that the more voltage is applied, the more power is available and the faster the motor can do work.

If a motor is under a fixed amount of load, and the voltage is increased (resulting in an increase of power), what will it do? It will spin faster! There is more power available to accomplish the same amount of work.

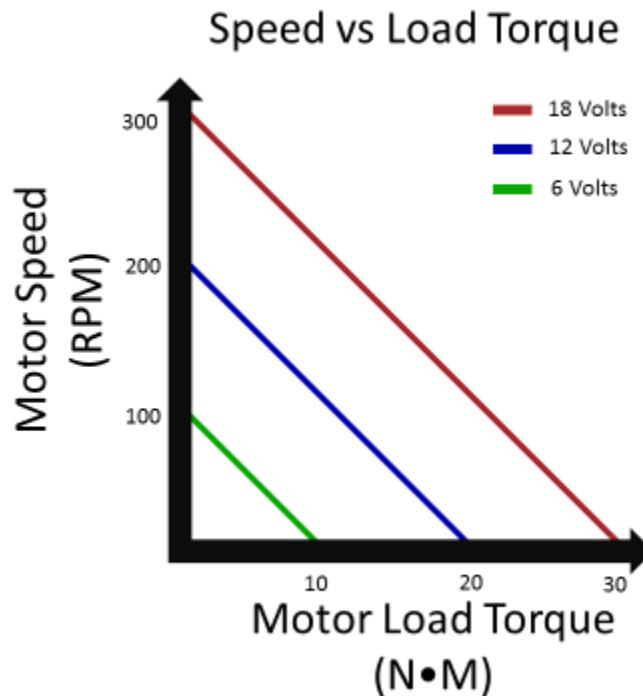
This means that the above motor characteristics change depending on the Voltage applied to the motor, and as such they need to be listed at a given Voltage (i.e. "Tested at 12V".) In fact, the four characteristics above vary proportionally with applied voltage. For example, if one knows a motor has a free speed of 50 RPM at 6V, if the voltage is doubled to 12V the free speed doubles also, and can be calculated to be 100 RPM.

One can calculate the value of one of these characteristics at a specific voltage if one knows the characteristic at another voltage by multiplying the known value by the ratio between the voltages. Note, this does not apply to the motor's Free Current, which is the same at any voltage.

$$\text{New Value} = \text{Spec Value} \times (\text{New Voltage} / \text{Spec Voltage})$$

For instance, in the example described above a motor's free speed is specified at 50 RPM at 6V. The designer is planning to run the motor at 8V – what will be the free speed at this voltage?

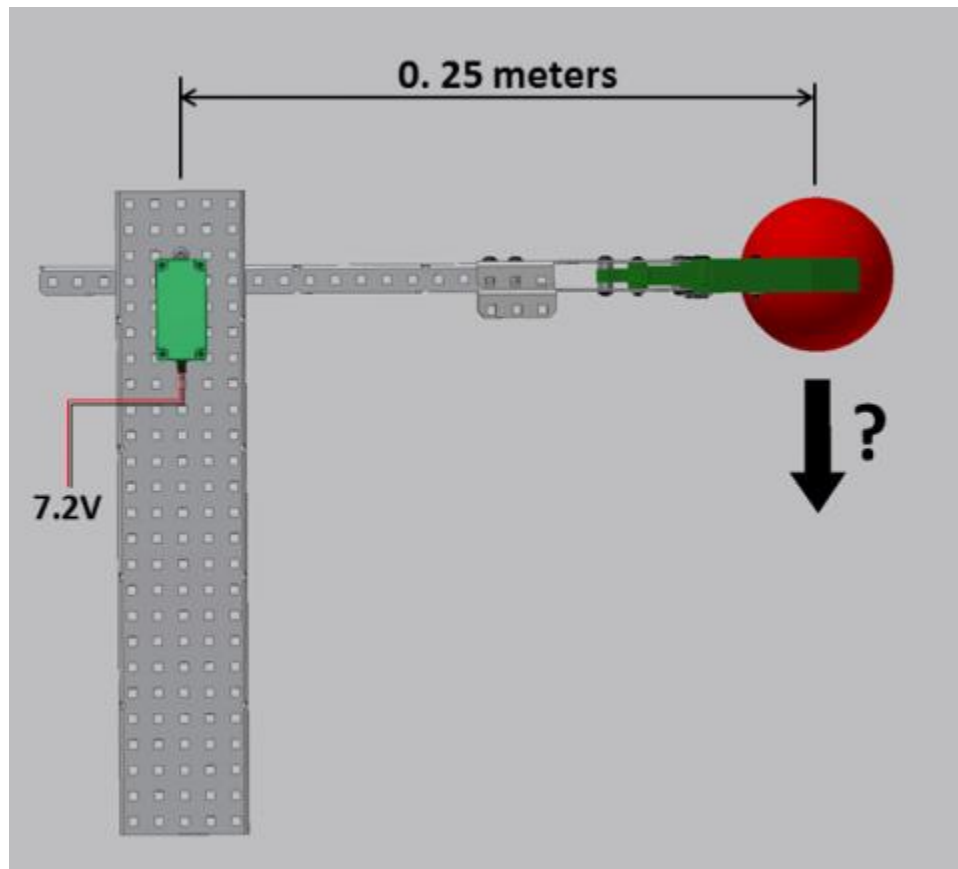
$$\text{Free Speed @ 8V} = \text{Free Speed @ 6V} \times (8\text{V} / 6\text{V}) = 50\text{RPM} \times (8/6) = 66.66 \text{ RPM}$$



So what implications does varying voltage have for control? The motors on a robot aren't just on / off devices. A robot designer can vary the voltage going to the motor under load to get different amounts of power, and varying speed. This is done using electrical devices known as motor controllers, which regulate the voltage supplied to the motors.

Motor Limits & Calculations

Does this mean a designer can keep adding voltage to a motor until it is capable of outputting the power needed for an application? Not quite, because motors have limits. At some point the power will be too much for the motor's electrical windings to handle and it will fail (usually, with a puff of white smoke.) Luckily, this doesn't happen with VEX motors – they all have internal thermal breakers that will cut the current being delivered to the motor if they get too hot. This is good because it means the motors won't burn out, but it presents a new constraint for designers: making sure the motors don't trip their breakers! How do designers do this? By designing their systems so that the motors don't draw more current than a specified amount by limiting the amount of load that will be applied.



Sample Motor Specs Tested at 7.2 Volts

Free Speed	100 RPM
Stall Torque	1.0 N•M
Free Current	0.1 Amps
Stall Current	3.0 Amps

Arm Load Calculation

In the above example, a known motor at a known voltage is driving a known robot arm. In this scenario, what is the maximum weight the robot can hold stationary?

To solve this problem, a designer must understand that the maximum weight the robot can hold stationary occurs at the stall torque of the motor. If the motor is stalled, it is applying a torque of 1 N-m on the robot arm, which is 0.25 meters long. Torque = Force * Distance

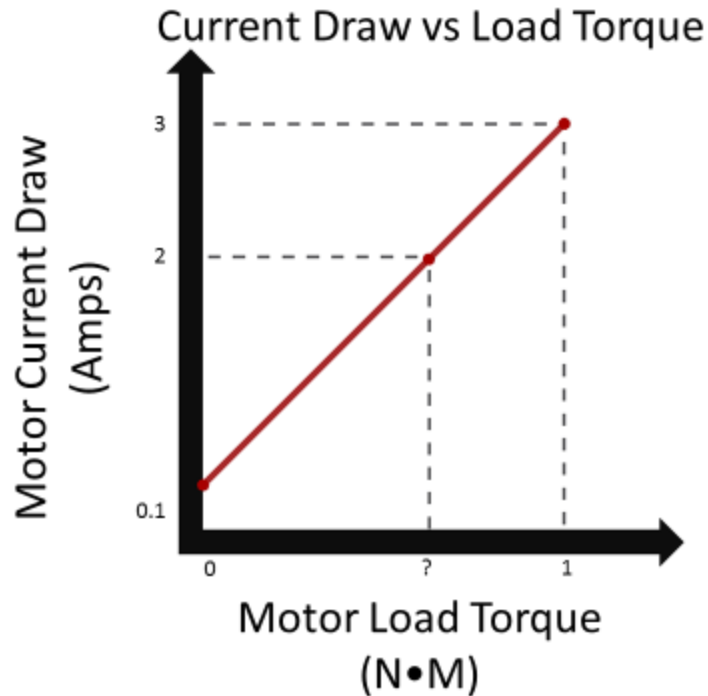
$$\text{Force} = \text{Torque} / \text{Distance} = 1 \text{ Newton-meter} / 0.25 \text{ meters} = 4 \text{ Newtons}$$

The arm can hold up 4 Newtons at motor stall. Any more and the arm will drop down.

Torque Load Calculated from Current Limit:

This is simple, but things get more complicated when one needs to consider a current limit. Say for instance in the above example a current limiting breaker is present in the motor that will trip if the motor draws more than 2 Amps. What is the maximum weight the robot can hold without tripping this breaker?

Now, the motor is not operating at stall torque – if the motor is at stall it will draw the stall current of 3 amps and trip the circuit breaker. The designer needs to figure out what torque load the motor must be under to maintain current draw less than 2 amps. How does one go about doing this?



Looking at the above graph and remembering that the relationships are linear, an equation can be derived to calculate the torque loading at any specified current draw.

The equation for a line is $y = mx + b$ where y is the value in the y -axis, x is the value in the x -axis, m is the slope of the line, and b is where the line intersects the y -axis (y -intercept).

The slope of the line can be expressed as $m = (\text{Change in } Y / \text{Change in } X) = (\text{Stall Current} - \text{Free Current}) / \text{Stall Torque}$

The Y -Intercept is the free current.

The Y value is the current at a given point on the line and the X value is the torque loading at that point.

The equation can be expressed as:

$$\text{Current} = ((\text{Stall Current} - \text{Free Current}) / \text{Stall Torque}) \times \text{Torque Load} + \text{Free Current}$$

Rearranging to solve for Torque Load:

$$\text{Torque Load} = (\text{Current} - \text{Free Current}) \times \text{Stall Torque} / (\text{Stall Current} - \text{Free Current})$$

Plugging the parameters of the above example in, one can solve for the torque load which results in a current draw of 2 Amps.

$$\text{Torque Load} = (2 \text{ Amps} - .1 \text{ Amps}) \times 1 \text{ N-m} / (3 \text{ Amps} - .1 \text{ Amps})$$

$$\text{Torque Load} = (1.9 \text{ Amps}) \times 1 \text{ N-m} / (2.9 \text{ Amps})$$

$$\text{Torque Load} = .655 \text{ N-m}$$

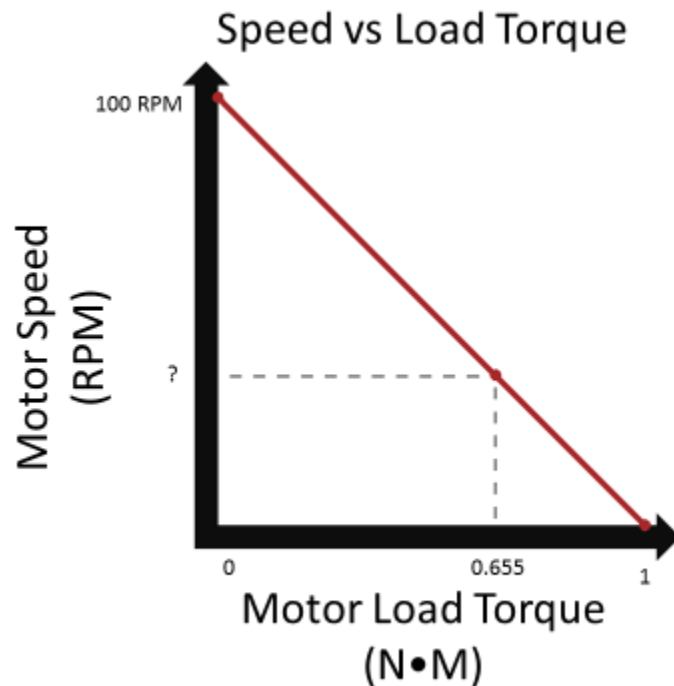
Based on this calculation, the designer knows that if the torque applied to the motor is above 0.655 N-m, the motor will exceed 2 amps of current draw, and the breaker will trip. All that is left is to calculate how much force can be placed on the arm.

$$\text{Force} = \text{Torque} / \text{Distance} = 0.655 \text{ N-m} / 0.25 \text{ m} = 2.62 \text{ N}$$

If the robot arm picks up an object heavier than 2.62 N, it will trip the motor circuit breaker.

Motor Speed from Torque Load Calculation

In the above example, how fast does the motor move when it is at the current limit? Based on the calculation in the previous step, the designer needs to determine the motor speed at a load of 0.655 N-m.



Looking at the above graph, it is possible to derive an equation to calculate the motor speed at any given torque load in a way similar to the current draw equation from the last example.

In this case, the slope of the line is expressed as $m = (\text{Change in Y}) / (\text{Change in X}) = -(\text{Free Speed}) / (\text{Stall Torque})$.

Note: the slope is negative.

The Y-Intercept is the Free Speed

The Y value is the speed at a given point on the line, and the X value is the load torque at that point.

The equation is expressed as:

$$\text{Speed} = -(\text{Free Speed} / \text{Stall Torque}) \times \text{Torque Load} + \text{Free Speed}$$

Plugging the parameters of the above example in, one can solve for the speed of the motor at a torque load of 6.55 in-lbs:

$$\text{Speed} = -(100 \text{ RPM} / 1 \text{ N-m}) \times 0.655 \text{ N-m} + 100 \text{ RPM}$$

$$\text{Speed} = -(100 \text{ RPM/N-m}) \times 0.655 \text{ N-m} + 100 \text{ RPM}$$

$$\text{Speed} = -65.5 \text{ RPM} + 100 \text{ RPM} = 34.5 \text{ RPM}$$

The motor will spin at 34.5 RPM when it is under a torque load of 0.655 N-m, while drawing 2 amps and lifting an object weighing 2.62 N.

Multiple Motors

When an application requires more power than a motor can handle, a designer has three options:

1. Deal with it, and change the requirements of the problem so that lower power is acceptable.
2. Switch to a more powerful motor.
3. Use more than one motor in the application.

What happens when multiple motors are used on one application? Simple – they balance the torque load between them. If 2 N-m of torque is applied, each motor will have a torque load of 1 N-m, and react accordingly.

A simple way to think about it is that the motors take on the characteristics of one super-motor with combined specs of the individual motors. The stall torques, stall currents, and free currents add together but the free speed doesn't change.

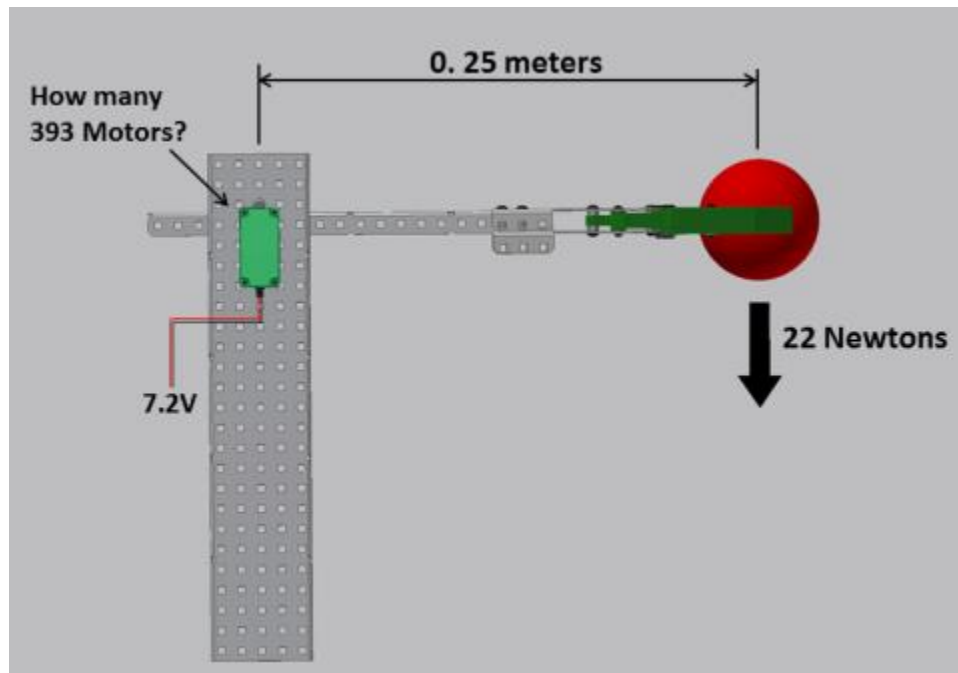
VEX 2-Wire Motor 393 (Single Motor)

	Default	High Speed
Free Speed	100 RPM	160 RPM
Stall Torque	1.67 N•M	1.04 N•M
Stall Current	4.8 Amps	4.8 Amps
Free Current	0.37 Amps	0.37 Amps

VEX 2-Wire Motor 393 (2 Motors)

	Default	High Speed
Free Speed	100 RPM	160 RPM
Stall Torque	3.34 N•M	2.08 N•M
Stall Current	9.6 Amps	9.6 Amps
Free Current	0.74 Amps	0.74 Amps

The above chart shows the specifications for the VEX 2-Wire Motor 393, and shows the specifications one would get when two are combined into the same application.



In the above example how many VEX 393 motors would it take to hold the object stationary?

The torque load on the motors can be calculated as follows:

$$\text{Torque Load} = \text{Force} \times \text{Distance} = 22 \text{ N} \times 0.25 \text{ m} = 5.5 \text{ N}\cdot\text{m}$$

Based on this torque load, one can compare it with the Stall Torque of the VEX 393 motor and determine how many would be needed.

$$5.5 \text{ N}\cdot\text{m} / 1.67 \text{ N}\cdot\text{m} = 3.29 \text{ motors}$$

Thus, it would take 4 motors to hold up the arm in the example above.

7.4: Arm Design

As part of the design of a competition robot, it is possible a designer would use an arm similar to the ones shown above to pick up objects and dump them in a goal. Students should apply the lessons learned in this unit to the following example:

1. Determine the total weight of the object manipulator designed in Unit 6.
2. Determine the weight of one game object for the game described in Unit 5.
3. Assuming a simple arm system where the arm is 0.25m long, calculate how many VEX 393 motors would be required to hold the arm stationary with the object manipulator and game object at its tip.

4. Assume the 393 motors cannot draw more than 2.5 amps before their internal circuit breakers will trip; calculate how many motors are required for this application, then calculate the rotational speed of the arm.

CONCLUSION

Reviewing the above example, one could say that this is not a practical design because of how many motors it takes. One variable the designer can change to require fewer motors to accomplish the same task is the length of the arm. A shorter arm would place less torque load on the motors as described in the section on Torque above. This is known as mechanical advantage or leverage. A short arm is not practical for many designs – are there other ways to utilize mechanical advantage so that less power can be used to do the same amount of work? In the next unit, designers will see how this is possible through the use of gear ratios.